Solving Systems of Equations

System of equations = consists of two or more linear equations with the same variables.

Ex) $\begin{cases} 2x+3y=4 \\ 3x-y=-5 \end{cases}$

Ex) $\begin{cases} x-y=1 \\ y=3 \end{cases}$

We will solve systems of equations by substitution.

Def: solution of a system = an ordered pair that satisfies both equations at the same time.

Decide whether an ordered pair is a solution to the system.

Ex) $\begin{cases} 3x-2y=-4 \\ 5x+y=15 \end{cases}$ $(2,5)$ → ordered pair

$$\begin{cases} 3x - 2y = -4 \\ 5x + y = 15 \end{cases} \quad (2, 5)$$
$x \quad y$

So, substitute "x" for "x" and "y" for "y"

Solution:

$3(2) - 2(5) = -4$ ✓

$6 - 10 = -4$

$-4 = -4$ ✓

$5(2) + 5 = 15$

$10 + 5 = 15$

$15 = 15$ ✓

YES!

BOTH equations must work!

Ex) Is $(1, -2)$ $\begin{cases} x - 3y = 7 \\ 4x + y = 5 \end{cases}$
$x \quad y$

$x - 3y = 7$

$1 - 3(-2) = 7$

$1 + 6 = 7$

$7 = 7$

$4x + y = 5$

$4(1) + (-2) = 5$

$4 - 2 = 5$

$2 \neq 5$

NO, not a solution!

Solve by Substitution:

$$\begin{cases} 2x+5y=7 \\ x=-1-y \end{cases}$$

$2(-1-y)+5y=7$

$-2-2y+5y=7$

$-2 \quad +3y=7$

$+2 \qquad +2$

$3y=9$

$y=3$

So,

$x=-1-3$

$x=-4$

$2x+5(3)=7$

$2x+15=7$

$\frac{2x}{2}=\frac{-8}{2}$

$x=-4$

Substitution

Solve: $\begin{cases} 3x + 5y = 26 \\ y = 2x \end{cases}$

Solution:

Use $y = 2x$ in place of y in the first equation of the system.

$$3x + 5(2x) = 26$$
$$3x + 10x = 26$$
$$\frac{13x}{13} = \frac{26}{13}$$
$$x = 2$$

So,

$$y = 2(2)$$
$$y = 4$$

$\boxed{(2, 4)}$

Ex) Solve by substitution

$$3x+2y=27$$
$$x=y+4$$

$$3(y+4)+2y=27$$
$$3y+12+2y=27$$
$$5y+12=27$$
$$-12\quad -12$$
$$\frac{5y}{5}=\frac{15}{5}$$
$$y=3$$

So, $x=3+4$

$$x=7$$

$$\boxed{(7,3)}$$

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Solve by substitution:

$$\begin{cases} 3x + 4 = -y \\ 2x + y = 0 \end{cases}$$

$$\frac{3x}{-1} + \frac{4}{-1} = \frac{-y}{-1}$$

$$-3x - 4 = y$$

$$2x + (-3x - 4) = 0$$

$$-1x - 4 = 0$$

$$+4 \quad +4$$

$$-1x = 4$$

$$x = -4$$

So, $2(-4) + y = 0$

$$-8 + y = 0$$

$$y = 8$$

$$\boxed{(-4, 8)}$$

$$\begin{cases} 4x+y=-2 & (1) \\ -2x-3y=1 & (2) \end{cases}$$

Solve by Substitution

$4x+y=-2$ Solve for y

$-4x \quad -4x$

$y=-4x-2$ → Substitute for "y"

$-2x-3(-4x-2)=1$

$-2x+12x+6=1$

$10x=-5$

$x=-\frac{1}{2}$

$4\left(-\frac{1}{2}\right)+y=-2$

$-2+y=-2$

$y=0$

$\left(-\frac{1}{2}, 0\right)$

Solving Systems of Equations by Elimination

Obj: Eliminate one variable to solve for the other.

Ex) Solve by elimination

$$\begin{array}{r} 3x - 1y = 7 \\ (+)\ 2x + 1y = 3 \\ \hline 5x \quad\quad = 10 \end{array}$$

$x = 2$

Substitute $x = 2$ in for "x" in the 2nd equation

So,

$2(2) + y = 3$

$4 + y = 3$

$y = -1$

$(2, -1)$

Ex) Solve by elimination

$$2x - y = 2$$
$$(+)\quad 4x + y = 10$$
$$6x = 12$$
$$x = 2$$

*Always look for variables with a coefficient of 1.

Sub $x = 2$ in for "x" in $2x - y = 2$

$$2(2) - y = 2$$
$$4 - y = 2$$
$$y = 2$$

#39 $3(4x + y = -2)3$ Solve by elimination

$$-2x - 3y = 1$$

$$12x + 3y = -6$$
$$(+)\quad -2x - 3y = 1$$
$$10x = -5$$
$$x = -1/2$$

So, $-2(-\frac{1}{2}) - 3y = 1$

$$+1 - 3y = 1$$
$$-3y = 0$$
$$y = 0$$

Ex) Solve by ~~Substitution~~ Elimination

$5(2x + 3y = -15)5$

$-2(5x + 2y = 1)-2$

Decide which variable you want to eliminate first!

$$10x + 15y = -75$$
$$+(-10x - 4y = -2)$$
$$11y = -77$$
$$y = -7$$

So,

$5x + 2(-7) = 1$

$5x - 14 = 1$

$+14 \quad +14$

$5x = 15$

$x = 3$

$\boxed{(3, -7)}$

Solve by Elimination

Ex) $4x - 3y = -19$

$3(2x + y = 13)3$

$\Downarrow$

$4x - 3y = -19$

$6x + 3y = 39$

$10x = 20$

$x = 2$

So,

$4(2) - 3y = -19$

$8 - 3y = -19$

$-8 \quad -8$

$-3y = -27$

$y = 9$

$(2, 9)$

#1

$$\begin{array}{r} 2x+5y=17 \\ +\ 6x-5y=-9 \\ \hline 8x=8 \end{array}$$

$$\frac{8x}{8}=\frac{8}{8}$$

$$x=1$$

So, $2(1)+5y=17$

$$2+5y=17$$

$$-2 \qquad -2$$

$$5y=15$$

$$\frac{5y}{5}=\frac{15}{5}$$

$$y=3$$

$$\boxed{(1,3)}$$

Bridge Math Warm Up 2/20/14

Solve by Elimination

$-8(6x + 7y = 4)$ -8

$7(5x + 8y = -1)$ 7

$\Downarrow$

$$-48x - 56y = -32$$
$$+\quad 35x + 56y = -7$$
$$-13x \qquad = -39$$

$$x = 3$$

So, $5(3) + 8y = -1$

$15 + 8y = -1$

$-15 \qquad -15$

$8y = -16$

$y = -2$

(49) $FG = 3x - 10$

$GH = 2x + 5$

$HF = x + 5$

$3x - 10 = 2x + 5$

$-5 \quad -5$

$3x - 15 = 2x$

$-3x \quad -3x$

$-15 = -1x$

$15 = x$

Solve by Elimination

(4)

$6x - y = -1$

$5y = 17 + 6x$

$\Rightarrow$

$6x - y = -1$

$-6x + 5y = 17$

$4y = 16$

$y = 4$

$5y = 17 + 6x$

$-6x \quad -6x$

$-6x + 5y = 17$

* Need in standard form

$5(4) = 17 + 6x$

$20 = 17 + 6x$

$-17 \quad -17$

$3 = 6x$

$\frac{1}{2} = x$

Pg 356 (1-6) ALL

(9-14) ALL

Types of Solutions

$0 = 0$

$2 = 2$

$7 = 7$

$\vdots$

Infinitely many solutions
(i.e. We have the same line)

$0 = 1$ **No solution**

$1 = 2$

$\vdots$

Eliminate "y" first.

#5 $2x+18y=-9$

$-1(4x+18y=-27)-1$

$2x+18y=-9$

$+\ -4x-18y=27$

$-2x=18$

$\frac{-2x}{-2}=\frac{18}{-2}$

$x=-9$

So, $2(-9)+18y=-9$

$-18+18y=-9$

$+18 \quad +18$

$18y=9$

$\frac{18y}{18}=\frac{9}{18}$

$y=\frac{1}{2}$

$\left(-9, \frac{1}{2}\right)$

Eliminate "x" first

$-2(2x+18y=-9)-2$

$4x+18y=-27$

$-4x-36y=18$

$+\ 4x+18y=-27$

$-18y=-9$

$y=\frac{1}{2}$

$\left(-9, \frac{1}{2}\right)$

So, $2x+18\left(\frac{1}{2}\right)=-9$

$2x+9=-9$

$-9 \quad -9$

$\frac{2x}{2}=\frac{-18}{2}$

$x=-9$