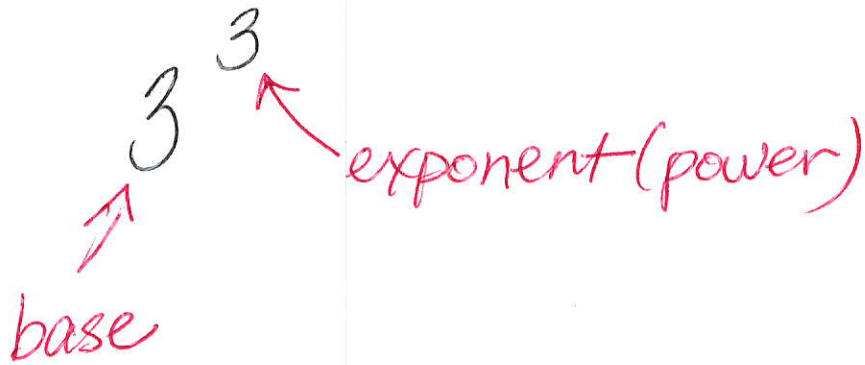
Exponents

- We use exponents to replace repeated multiplication.

For example:

$$3 \cdot 3 \cdot 3 = 3^3 = 27$$



Rules of Exponents

① Product Rule: $a^m \cdot a^n = a^{m+n}$

Keep the base, add the exponents.

② Power Rule: $(a^m)^n = a^{mn}$

Keep the base, multiply the exponents

Power Rule: $(a \cdot b)^m = a^m b^m$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

- Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

  Keep the base, subtract the exponents

- Negative to Positive: $a^{-n} = \frac{1}{a^n}$

- Anything to the zero equals 1: $a^0 = 1$

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

Exponents Day 1

Ex) Simplify: $3^2 \cdot 3^3 = 3^5 = 243$

Ex) Simplify: $(-7)^3 \cdot (-7)^4 = (-7)^7$

$= -823543$

Ex) Simplify: $z^2 \cdot z^5 \cdot z^6 = z^{13}$

Ex) $5m^2 \cdot 2m^6 = (5 \cdot 2)(m^8) = 10m^8$

*Side note: Variables should appear <u>only</u> once.

Ex) Simplify: $(2a^4b^4)^4$ ← Apply the Power Rule

$\Downarrow$

$2^4 a^4 b^4 = 16a^4b^4$

Ex) Simplify: $(3a^2b^4)^5$

Goal: Get rid of parentheses by applying the Power Rule.

$(3a^2b^4)^5 = 3^5a^{10}b^{20} = 243a^{10}b^{20}$

Multiply the powers.

Ex) Simplify $\left(-\frac{2x}{5}\right)^3 = -\frac{2^3x^3}{5^3} = -\frac{8x^3}{125}$

Ex) Simplify: $(2wx^3y^2)^2(x^4y^5) =$

$2^2w^2x^6y^2 \cdot x^{20}y^5 =$

$4w^2x^6 \cdot x^{20} \cdot y^2 \cdot y^5 =$

$4w^2x^{26}y^7$

~~③ $2v^{-2} \cdot 3u^2$~~

① $2yx^2 \cdot 4xy^2 \cdot 3x^4y^2 =$

$(2 \cdot 4 \cdot 3)(x^2 \cdot x^1 \cdot x^4)(y^1 \cdot y^2 \cdot y^2) =$

$24x^7y^5$

③ $2v^{-2} \cdot 3u^2 =$

$(2 \cdot 3) \cdot v^{-2} \cdot u^2 =$

$6v^{-2}u^2 =$

$\frac{6}{1} \cdot \frac{1}{v^2} \cdot \frac{u^2}{1} =$

$\frac{6u^2}{v^2}$

⑤ $u^{-4}v^2 \cdot 4u^2v^2 =$

$4(u^{-4} \cdot u^2)(v^2 \cdot v^2) =$

$4u^{-2} \cdot v^4 = \frac{4v^4}{u^2}$

#7 $x^0 y^{-4} \cdot 4x^2 y^3 =$ Simply. Write using only positive ~~expo~~ integer exponents.

$y^{-4} \cdot 4x^2 y^3 =$

$4x^2 y^{-1} = \frac{4x^2}{y}$

In class

Simplify

① $(5r^3t^2)^4 = 5^4 r^{12} t^8 = 625 r^{12} t^8$

② $(-2w^3z^7)^4 = (-2w^3z^7)(-2w^3z^7)(-2w^3z^7)(-2w^3z^7)$

$= 16w^{12}z^{28}$

③ $\left(\frac{-2a}{b^2}\right)^7 = \frac{-2^7 a^7}{b^{14}} = -\frac{128a^7}{b^{14}}$

④ $-\left(\frac{2a^3c}{5b^2}\right)^5 = -\frac{32a^{15}c^5}{3125b^{10}}$

⑤ $(-13g)^3(-13g)^1 = (-13g)^4 = 28561g^4$

OR

$-2197g^3 \cdot (-13g) = 28561g^4$

Bridge Math HW 3/20/14

① $\left(\frac{xy}{z^2}\right)^4$

② $(-6p)^4(-6p)$

③ $\left(\frac{6x^3y^9}{z^5}\right)^4$

④ $(2x^2y^3z)^4(xy^2z^3)^2$

⑤ $\left(\frac{1}{5}\right)^4(2x)^2$

#2 $(-6p)^4(-6p)^1 =$

$(-6p)^5 =$

$-6^5p^5 = -7776$

$= -7,776p^5$

#3 $\left(\frac{6x^3y^9}{z^5}\right)^4 =$

$\frac{6^4x^{12}y^{36}}{z^{20}} = \frac{1296x^{12}y^{36}}{z^{20}}$

#5 $\left(\frac{1}{5}\right)^4(2x)^2 = \frac{1}{5^4} \cdot \frac{2^2x^2}{1}$

$= \frac{1}{625} \cdot \frac{4x^2}{1} = \frac{4x^2}{625}$

Exponents (Using the Quotient Rule)

Simplify. Write using only positive exponents.

1) $\dfrac{x^{-3}}{y^{-2}} = \dfrac{\frac{1}{x^3}}{\frac{1}{y^2}} = \dfrac{1}{x^3} \cdot \dfrac{y^2}{1} = \dfrac{y^2}{x^3}$

2) $\dfrac{7^{-1}}{5^{-4}} = \dfrac{\frac{1}{7}}{\frac{1}{5^4}} = \dfrac{1}{7} \cdot \dfrac{5^4}{1} = \dfrac{625}{7}$

3) $\dfrac{4h^{-5}}{m^{-2}k} = \dfrac{4}{1} \cdot \dfrac{1}{h^5} \cdot \dfrac{\frac{1}{1}}{\frac{1}{m^2}} \cdot \dfrac{1}{k} = \dfrac{4m^2}{h^5k}$

$= 1 \cdot \dfrac{m^2}{1}$

4) $\dfrac{a^{-2}b}{3d^{-3}} = \dfrac{1}{3} \cdot \dfrac{1}{a^2} \cdot \dfrac{b}{1} \cdot \dfrac{1}{\frac{1}{d^3}} = \dfrac{1bd^3}{3a^2}$

5) $\left(\frac{x}{2y}\right)^{-4} = \frac{x^{-4}}{2^{-4}y^{-4}} =$

Quotient Rule (continued)

$$\frac{a^m}{a^n} = a^{m-n}$$ "Keep the base, subtract the exponents".

Simplify

Ex) $\frac{6^{-3}}{6^2} = 6^{-5} = \frac{1}{6^5} = \frac{1}{7776}$

Ex) $\frac{4^{-3}}{5^{-2}} = \frac{\frac{1}{4^3}}{\frac{1}{5^2}} = \frac{1}{4^3} \cdot \frac{5^2}{1} = \frac{25}{64}$

Ex) $\frac{6^4 x^8}{6^5 x^3} = \frac{1}{6^1} \cdot \frac{x^5}{1} = \frac{x^5}{6}$

Ex) $\frac{x^{-3} y}{4z^{-2}} = \frac{yz^2}{4x^3}$ $\frac{\frac{1}{x^3} \cdot \frac{y}{1}}{4 \cdot \frac{1}{z^2}} = \frac{\frac{y}{x^3}}{\frac{4}{z^2}} = \frac{y}{x^3} \cdot \frac{z^2}{4} = \frac{yz^2}{4x^3}$

Practice and Problem Solving

Simplify. Write using only positive exponents.

① $\frac{3^8 y^5}{3^{10} y^2}$

② $\frac{6y^3}{2y}$

③ $\frac{10p^8}{2p^4}$

④ $\frac{p^{-5} q^{-4}}{9r^{-3}}$

⑤ $x^{-3} \cdot x^5 \cdot x^{-4}$

⑥ $\frac{(3x)^{-2}}{(4x)^{-3}}$

⑦ $\left(\frac{p^{-4} q}{r^{-3}}\right)^{-3}$

⑧ $\frac{(m^8 n^{-4})^2}{m^{-2} n^5}$

⑨ $\frac{-3k^5}{(2k)^2}$

⑩ $\left(\frac{4xy^2}{x^{-1} y}\right)^{-2}$

Selected Solutions

#4 #9 #10

#4

$$\frac{p^{-5}q^{-4}}{9r^{-3}} = \frac{\frac{1}{p^5q^4}}{9 \cdot \frac{1}{r^3}} = \frac{1}{p^5q^4} \cdot \frac{r^3}{9} = \frac{r^3}{9p^5q^4}$$

$$\Downarrow$$

$$\frac{9}{r^3}$$

#9

$$\frac{-3k^5}{(2k)^2} = \frac{-3k^5}{2^2k^2} = \frac{-3k^3}{4}$$

No ( ) here, so we can't apply the power rule.

#10

$$\left(\frac{4xy^2}{x^{-1}y}\right)^{-2} = \frac{4^{-2}x^{-2}y^{-4}}{x^2y^{-2}} = 4^{-2}x^{-4}y^{-2}$$

$$= \frac{1}{4^2x^4y^2}$$

$$= \frac{1}{16x^4y^2}$$

#8 $\dfrac{(m^8 n^{-4})^2}{m^{-2} n^5} = \dfrac{m^{16} n^{-8}}{m^{-2} n^5} = \dfrac{\frac{m^{16}}{n^8}}{\frac{n^5}{m^2}}$

$= \dfrac{m^{16}}{n^8} \cdot \dfrac{m^2}{n^5}$

$\dfrac{m^{16}}{1} \cdot \dfrac{1}{n^8}$

$= \dfrac{m^{18}}{n^{13}}$

#6 $\dfrac{(3x)^{-2}}{(4x)^{-3}} = \dfrac{3^{-2} x^{-2}}{4^{-3} x^{-3}} = \dfrac{\frac{1}{3^2 x^2}}{\frac{1}{4^3 x^3}}$

$= \dfrac{1}{9x^2} \cdot \dfrac{64x^3}{1}$

$= \dfrac{64x^3}{9x^2} = \dfrac{64x}{9}$

Exponents

Simplify: Write using only positive exponents

① $\left(\frac{rs^2t^3}{3t^4}\right)^6$

② $(-2ab^3c)^4(-2a^2b)^3$

*③ $(4x^{-2}y^{-3})^{-2}$

④ $\left(\frac{k^4t^2}{k^2t^{-4}}\right)^{-2}$

*⑤ $\frac{(3^{-1}x^{-3}y)^{-1}(2x^2y^{-3})^2}{(5x^{-2}y^2)^{-2}}$

Solutions

(#3) $(4x^{-2}y^{-3})^{-2} = 4^{-2}x^{4}y^{6}$

$= \dfrac{x^4y^6}{4^2}$

$= \dfrac{1x^4y^6}{16}$

these are equivalent

$= \dfrac{1}{16}x^4y^6$

(#5) $\dfrac{(3^{-1}x^{-3}y)^{-1}(2x^2y^{-3})^2}{(5x^{-2}y^2)^{-2}} =$

$\dfrac{3^1x^3y^{-1}\cdot 2^2x^4y^{-6}}{5^{-2}x^4y^{-2}}$

$$= \frac{12 x^{7} y^{-7}}{\frac{1}{25} \cdot x^{4} \cdot \frac{1}{y^{2}}}$$

$$= 300 x^{3} y^{-5}$$

$$= \frac{300 x^{3}}{y^{5}}$$